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**ENERGY-CONSISTENT MULTISCALE ALGORITHMS FOR GRANULAR FLOW**

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**CALIFORNIA INSTITUTE OF TECHNOLOGY**

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<b>14. ABSTRACT</b> <p>In this final report, we document the achievements made as a result of this Young Investigator Program (YIP) project. We worked on the development of multi scale energy-consistent algorithms to simulate and capture flow phenomena in granular materials. For this, we have made progress in three key areas: i) the development of unprecedented algorithms at the grain scale to simulate flow of complex (real) granular media; ii) the development of multiscale and continuum approaches to simulate the continuum behavior of granular media under large flow; iii) the development of experimental techniques and approaches to model the behavior of granular materials under extreme avalanche flow. In the area of algorithmic development at the grain scale, we have successfully developed a technique, called the granular element method (GEM), and have been able to bypass all the major shortcomings of current discrete element methods (DEM). We have shown that the technique is not only much more efficient algorithmically, but displays a degree of accuracy that is unprecedented for discrete methods. In the area of multiscale and continuum approaches, we have focused our attention on the development of continuum models that are micro-inspired and in this way can bypass much of the</p>					
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# ENERGY-CONSISTENT MULTISCALE ALGORITHMS FOR GRANULAR FLOW

FA9550-11-1-0052

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## **Abstract**

In this final report, we document the achievements made as a result of this Young Investigator Program (YIP) project. We worked on the development of multi scale energy-consistent algorithms to simulate and capture flow phenomena in granular materials. For this, we have made progress in three key areas: i) the development of unprecedented algorithms at the grain scale to simulate flow of complex (real) granular media; ii) the development of multiscale and continuum approaches to simulate the continuum behavior of granular media under large flow; iii) the development of experimental techniques and approaches to model the behavior of granular materials under extreme avalanche flow. In the area of algorithmic development at the grain scale, we have successfully developed a technique, called the granular element method (GEM), and have been able to bypass all the major shortcomings of current discrete element methods (DEM). We have shown that the technique is not only much more efficient algorithmically, but displays a degree of accuracy that is unprecedented for discrete methods. In the area of multiscale and continuum approaches, we have focused our attention on the development of continuum models that are micro-inspired and in this way can bypass much of the phenomenology that plagues current continuum models for granular flow. Finally, in the areas of experimental techniques, we have been able to use clever flow experiments in tandem with our advanced models to decipher the mechanisms controlling the main continuum features observed previously but that have not been able to be captured comprehensively in models. The consequences of these advancements are broad and deep. The GEM method has revolutionized the accuracy of discrete methods: capturing real material behavior for the first time. The multiscale continuum models have been able to explain the source of rate dependence in frictional resistance and, in conjunction with the experiments, have been able to decode the lifecycle of granular avalanches.

## **Status/Progress**

In this grant, we have focused mainly in making progress within three (3) areas of mayor interest: (1) a new simulation platform called the granular element method (GEM); (2) new multiscale continuum models to simulate the behavior of granular flows; (3) advanced experiments to look into the mechanism controlling granular flows and use them in tandem with the models to understand the lifecycle of granular avalanches.

One of the main features of the newly proposed GEM is that it can capture real particle shapes, which are quite complex. From a computational standpoint, this revolutionizes discrete models by giving the method a tool based on Non-Uniform Rational B-Splines

(NURBS). Once NURBS are introduced, the algorithm to detect contact needs to be redesigned to be able to detect contact points, even in non-convex surfaces. To achieve this, we developed an algorithm based on supporting separating hyperplanes, as the one shown schematically in Figure 1 and presented in [1]. This procedure has also been extended to perform calculations in non-convex surfaces without any restriction in relation to shape. This clearly revolutionizes a method that has been around for more than 3 decades and now, for the first time, can directly account for complex particle shapes, including non-convexity.

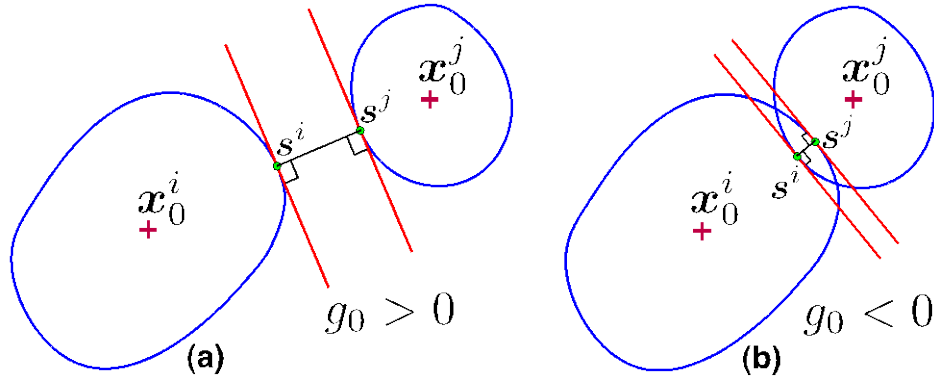


Figure 1. Supporting separating hyperplanes for two convex particles: (a) positive separation and (b) negative separation. Figure taken from [1].

The algorithmic developments achieved with GEM have enabled us to perform simulations of granular flows as shown in Figure 2. These simulations have shown, conclusively, the extremely important effect of particle shape in the overall strength of particle masses. Figure 2 shows a comparison of two identical column drop experiments, with the only difference being the particle shape, where the figures on the left show results with discs and the figures to the right show the same simulation with angular particles. At the end of the simulation, the array with particle shape is able to sustain an angle of repose of several degrees, whereas the simulation with discs has completely melted to a flat surface of a few particles in thickness. This result is extremely important for many applications, including terrestrial and extraterrestrial geosciences where the angle of repose controls many of the geomorphological processes. In engineering also the shape and texture of microstructure is usually very important, including polycrystalline metals and biomedical devices.

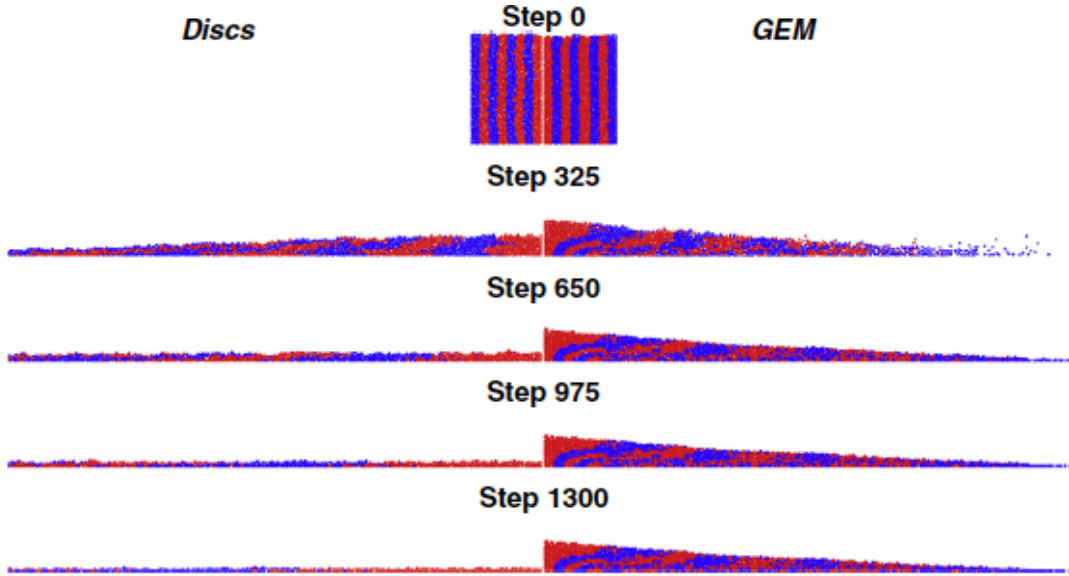


Figure 2. Collapse of granular column highlighting effect of particle shape. Left: spherical particles. Right: angular particles. New Granular Element Method (GEM) can capture this important shape effect, which accounts for enormous differences in angle of repose. Figure taken from [1].

The discrete simulations have been utilized beyond the algorithmic development dimension to shed light into the physics of granular materials under flow. Specifically, unraveling the rate-dependence of frictional resistance has derived multiscale constitutive models. It is well known that granular materials are pressure-dependent and the Mohr-Coulomb model has been used as the main framework to capture such dependence. However, the frictional dependence of the material has typically been treated as a constant, and certainly rate-independent. The theory of plasticity for granular material has been based on rate-independence of frictional strength. However, physical tests under flow have shown strong rate dependence in granular materials under steady-state. In our work, we have developed numerical models such as the one shown in Figure 3. Using this model, we have been able to discover an inherent scaling of collisional dissipation rates and dilatancy which, when combined, give rise to rate-strengthening [2].

Finally, an area of focus has been to put into use the models developed in [2], to gain further understanding in granular avalanches. Figure 4 shows how our rate-dependent model is able to capture and explain the lifecycle of granular avalanches. Experiments in the rotating drum configuration have been performed and have shown a clear difference in energy (and angle) between the angle of avalanche and the natural angle of repose. It has been known that this difference can be of close to 10 degrees. With our theoretical model, we are able to explain that the cause for this, previously unexplained, difference is exactly accounted for by the development of granular dilatancy. Showing that experiments combined with theoretical models can further increase our understanding of physical phenomena. We have reported these results in [3].

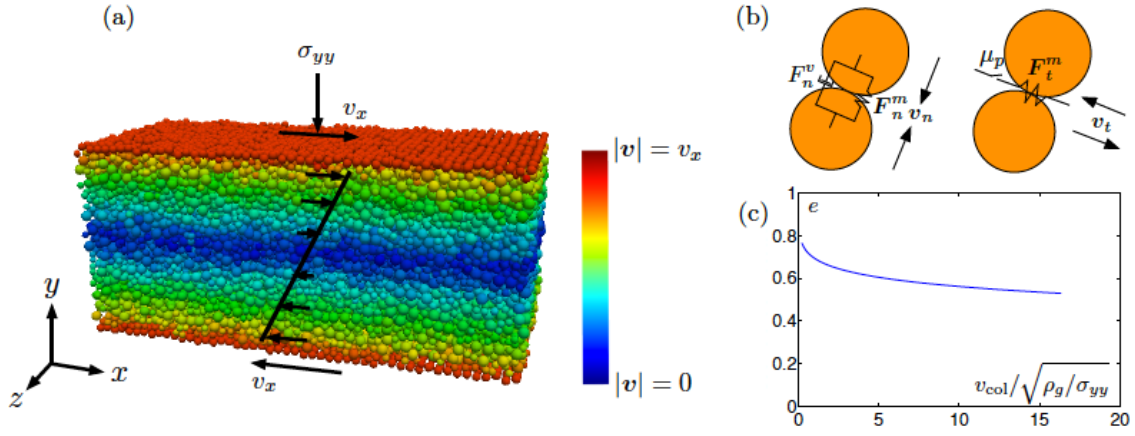


Figure 3. (a) Shear flow between two walls with steady-state velocity profile; (b) Illustration of the contact law; (c) Coefficient of restitution  $e$  in simulated system as a function of initial collision velocity. Figure taken from [2].

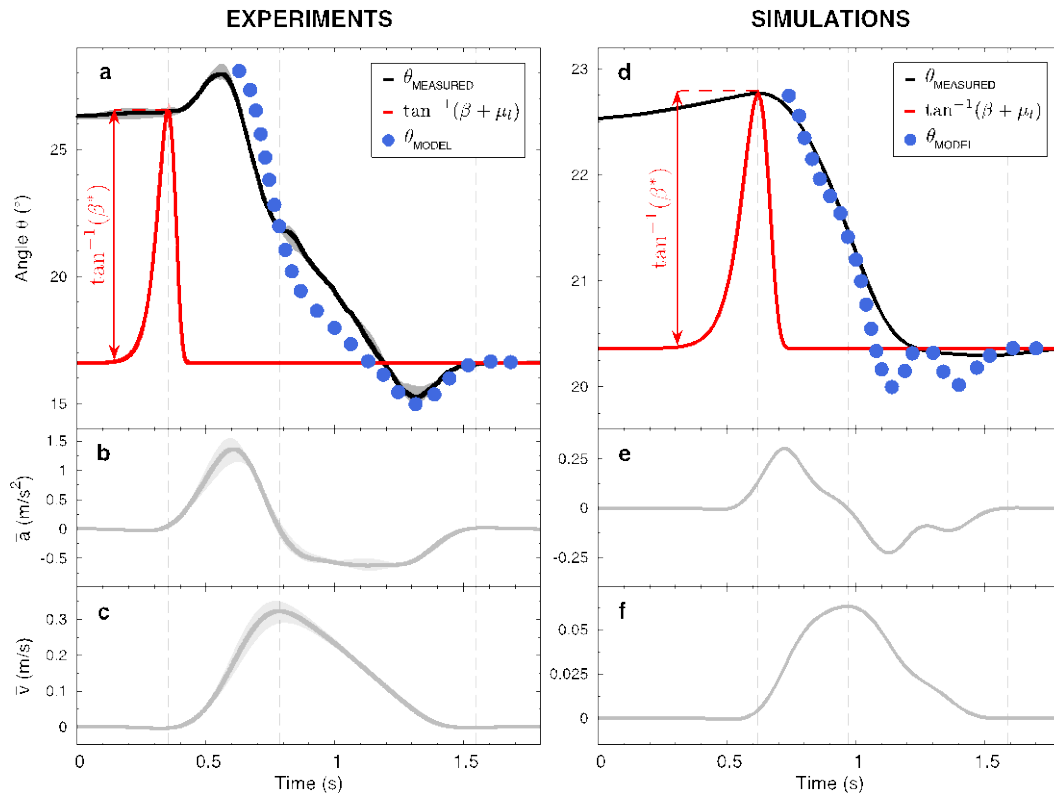


Figure 4. Evolution of the inclination angle and kinematics as a function of time and comparison with the theoretical model for (Left) the experiments and (Right) simulations using discrete models. Figure taken from [3].

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### **References**

1. K. W. Lim, K Krabbenhoft and JE Andrade, On the contact treatment of non-convex particles in the granular element method, Computational Particle Mechanics, in press, 2014.
2. R. C. Hurley, J. E. Andrade. Friction in inertial granular flows: Competition between dilatancy and microscopic dissipation rates. *In review*.
3. E. Marteau, R. C. Hurley, J. E. Andrade. Investigating the life-cycle of laboratory avalanches. *In review*.

### **Personnel Supported During Duration of Grant**

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Keng-Wit Lim	Ph.D. Candidate, Caltech, Pasadena, CA
Ryan Hurley	Ph.D. Candidate, Caltech, Pasadena, CA

### **Publications**

- K. W. Lim, K Krabbenhoft and JE Andrade, On the contact treatment of non-convex particles in the granular element method, Computational Particle Mechanics, in press, 2014.
- R. C. Hurley, J. E. Andrade. Friction in inertial granular flows: Competition between dilatancy and microscopic dissipation rates. *In review*.
- R. C. Hurley, J. E. Andrade. The origin of macroscopic friction and rate-dependence in dense granular flows. Engineering Mechanics Institute (EMI) Conference. Evanston, IL. Oral and paper presentation. 2013.
- KW Lim, K Krabbenhoft and JE Andrade, A contact dynamics approach to the granular element method, Computer Methods in Applied Mechanics and Engineering, 268, 557-573, 2014. *International Conference of the Engineering Mechanics Institute*, Minnesota, MN, 2008.

### **Honors & Awards Received**

- ASCE's Arthur Casagrande Professional Development Award, 2011
- AFOSR Young Investigator (YIP) Award, 2010
- Dr. Vicente Rocafuerte medal in science and research from the Republic of Ecuador, 2011
- National Research Council Committee for on State of the Art & Practice for Earthquake-Induced Liquefaction Assessment, 2013

### **AFRL Point of Contact**

None.

### **Transitions**

None.

### **New Discoveries**

- We have discovered that the angle of repose is heavily controlled by particle morphology or shape
- We have discovered an inherent scaling of collisional dissipation rates and dilatancy which, when combined, give rise to rate-strengthening
- We have discovered that the difference between the angle of avalanche and the angle of repose is furnished by the dilatancy development